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Heat and mass transfer between a rough wall and turbulent fluid flow at high Reynolds and Péclet numbers

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General dimensional and similarity arguments are applied to derive a heat and mass transfer law for fully turbulent flow along a rough wall. The derivation is quite analogous to Millikan's (1939) derivation of a skin-friction law for smooth- and rough-wall flows and to the derivation of the heat and mass transfer law for smooth-wall flows by Fortier (1968*a*, *b*) and Kader & Yaglom (1970, 1972).

The equations derived for the heat or mass transfer coefficient (Stanton number) C_h and Nusselt number Nu include the constant term β of the logarithmic equation for the mean temperature or concentration of a diffusing substance. This term is a function of the Prandtl number, the dimensionless height of wall protrusions and of the parameters describing the shapes and spatial distribution of the protrusions. The general form of the function β is roughly estimated by a simplified analysis of the eddy-diffusivity behaviour in the proximity of the wall (in the gaps between the wall protrusions). Approximate values of the numerical coefficients of the equation for β are found from measurements of the mean velocity and temperature (or concentration) above rough walls. The equation agrees satisfactorily with all the available experimental data. It is noted that the results obtained indicate that roughness affects heat and mass transfer in two ways: it produces the additional disturbances augmenting the heat and mass transfer and simultaneously retards the fluid flow in the proximity of the wall. This second effect leads in some cases to deterioration of heat and mass transfer from a rough wall as compared with the case of a smooth wall at the same values of the Reynolds and Prandtl numbers.

1. Introduction

At least several dozen published papers are devoted to the experimental investigation of turbulent heat transfer in pipes with rough walls (some of them will be referred to and analysed below). Recently some studies were also published which contain a theoretical analysis of heat and mass transfer between a rough wall and a turbulent fluid flow based on specific eddy-viscosity and eddy-thermal-diffusivity assumptions (e.g. Jayatilleke 1969; Millionshchikov

1971). In this paper another approach to the problem will be developed; it is applicable at high enough Reynolds and Péclet numbers and is based primarily on general dimensional and similarity arguments. This approach is quite analogous to the well-known method of Izakson (1937) and Millikan (1939) for deriving the logarithmic velocity profile and logarithmic skin-friction law of Prandtl and Nikuradse for turbulent flows in pipes and channels. The same method was applied recently by Csanady (1967) and Gill (1967) to the derivation of the skin-friction law for the turbulent Ekman boundary layer in a rotating fluid, and by Fortier (1968*a*, *b*) and Kader & Yaglom (1970, 1972) to the derivation of the heat and mass transfer law for turbulent flows along a smooth wall (see also Monin & Yaglom 1971, §§ 5.7 and 6.6). The close relation of this method to the general 'asymptotic matching' principle was analysed by Blackadar & Tennekes (1968), Tennekes (1968), Yajnik (1970) and Afzal & Yajnik (1971).

It is also possible to apply similar arguments in the investigation of the law of heat and mass transfer between a rough wall and a fully turbulent fluid flow (this possibility was emphasized, for example, by Fortier 1968*b*). However, the unknown value of one particular function of several variables must be determined in order to obtain an explicit expression for heat and mass transfer coefficients in such a way. The approximate evaluation of this function with the aid of existing data and some rough physical reasoning will be considered in § 2 of this paper. The value found for this function implies quite definite predictions for the law of heat and mass transfer between a rough wall and a fully turbulent fluid flow. The law so derived will be compared in the closing section of the paper with numerous experimental data available on turbulent heat transfer in rough pipes.

2. Heat and mass transfer law for flow along a completely rough wall

Let us consider turbulent parallel flow along a homogeneous rough wall in a plane channel, circular pipe or boundary layer along a flat plate without a longitudinal pressure gradient. We shall assume that the wall has a constant temperature θ_w different from the temperature of the fluid.† Then the mean temperature profile $\theta(y)$ near the wall (but above the tops of wall protrusions) will be described by the general temperature wall law

$$\theta_w - \theta(y) = \theta_* \phi(y_+, Pr, h_+, \sigma_1, \sigma_2, \ldots), \tag{1}$$

where $\theta_* = j_w/c_p \rho u_*$ is the so-called heat-flux temperature (or friction temperature), $u_* = (\tau_w/\rho)^{\frac{1}{2}}$ the friction velocity, τ_w and j_w wall values of the shear stress

† For definiteness we shall henceforth talk, as a rule, about heat transfer only and therefore call the quantity θ temperature. However, all the subsequent reasoning can also be applied to mass transfer. In this case θ should of course be the concentration of the transferred substance and correspondingly j_w and χ should be a wall mass flux and a molecular mass diffusivity, while c_p must be replaced by one.

Let us also note that the condition θ_w = constant may be replaced by j_w = constant since such a replacement does not change the mass transfer in all the cases with the exception of the case of $Pr \ll 1$, which we shall not consider below (cf., for example, Siegel & Sparrow 1960; Kays 1966).

and heat flux, ρ and c_p the fluid density and its specific heat, $y_+ = hu_*/\nu$, $h_+ = hu_*/\nu$, y the distance from the wall, h the mean height of roughness elements ('wall protrusions'), $Pr = \nu/\chi$ the Prandtl number, ν and χ the molecular viscosity and molecular thermal diffusivity and $\sigma_1, \sigma_2, \ldots$, dimensionless parameters which characterize the shapes of roughness elements, their distribution on the wall and (in cases when not all the roughness elements are identical) the scatter of their sizes and shapes. (H. B. Squire (1951) was apparently the first to state this law for the case of a smooth wall.) Similarly, the mean temperature profile in the core of the flow (or in the outer part of the boundary layer) is described by the temperature defect law if the numbers $Re = LU/\nu$ and $Pe = LU/\chi = RePr$ are high enough, where L and U are typical length and velocity scales of the flow. This defect law has the form

$$\theta(y) - \theta_1 = \theta_* \phi_1(\eta), \tag{2}$$

where $\theta_1 = \theta(L)$, $\eta = y/L$, and L is the half-width of the channel, pipe radius or boundary-layer thickness. (This law was first stated by W. Squire in 1959 in a slightly different form.) Now let us assume that the Reynolds and Péclet numbers are also so high that not only are laws (1) and (2) valid for corresponding values of y but there is also an 'overlap interval' of y values in which both the laws apply simultaneously. Then the arguments which were formulated by Izakson (1937) for the turbulent velocity profile can be applied to the temperature profile too. These arguments imply that both the functions ϕ and ϕ_1 must be logarithmic in the overlap interval:

$$\left.\begin{aligned} \phi(y_+, Pr, h_+, \sigma_1, \sigma_2, \ldots) &= \alpha \ln y_+ + \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots), \\ \phi_1(\eta) &= -\alpha \ln \eta + \beta_1 \end{aligned}\right\} \tag{3}$$

(cf. Monin & Yaglom 1971, §§ 5.5 and 5.7). If we now substitute (3) in (1) and (2) and then add the results we obtain

$$c_h = \frac{(\tfrac{1}{2}c_f)^{\frac{1}{2}}}{\alpha \ln [Re(\tfrac{1}{2}c_f)^{\frac{1}{2}}] + \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots) + \beta_1}, \tag{4}$$

where $c_h = j_w/c_p \rho U(\theta_w - \theta_1)$ is the heat transfer coefficient (Stanton number) and $c_f = 2(u_*/U)^2$ is the skin-friction coefficient. Equation (4) represents a general law for heat (or mass) transfer between a rough wall and a turbulent fluid flow.

The numerical coefficients α and β_1 in (3) and (4) do not depend on the specific features of the wall, i.e. they are the same for any smooth or rough wall. The determination of their values from the available data was discussed in detail by Kader & Yaglom (1972) in relation to the problem of the turbulent heat and mass transfer law for smooth-wall flows. According to the information contained in the cited paper the available measurements of the temperature profile and the wall heat flux j_w in various turbulent wall flows imply the estimate $\alpha \approx 2{\cdot}12$. The accuracy of this estimate is close to the accuracy of the well-known estimate $A = 1/k \approx 2{\cdot}5$, i.e. $k \approx 0{\cdot}4$, where A is the leading coefficient in the logarithmic equation for the mean velocity and k is von Kármán's constant. The value of the coefficient β_1 will be different for flows in circular pipes, plane channels and boundary layers along a flat plate. For pipe and channel flows β_1 is relatively close to zero. Further, we shall follow Kader & Yaglom (1972) and assume that

$\beta_1 \approx 0{\cdot}5$, but experimental evidence supporting such a choice is not very reliable and therefore at present it is also possible to assume that $\beta_1 = 0$ for pipe and channel flows.† Moreover there were no data at all on the value of β_1 for boundary-layer flows until quite recently. Therefore Kader & Yaglom (1972) recommended use of the value $\beta_1 \approx 2{\cdot}35$, which follows from the available data on the velocity defect in the outer part of a turbulent boundary layer if Reynolds' analogy is supposed to be valid. This preliminary estimate turns out to agree quite well with the recent direct measurements by Šlančiauskas, Pedišius & Žukauskas (1971) and by Šlančiauskas & Drižius (1971). As regards the value of the coefficient $\beta(Pr, h_+, \sigma_1, \sigma_2, \ldots)$, it depends essentially on the wall roughness and therefore the corresponding smooth-wall estimate of Kader & Yaglom (1970, 1972) cannot be applied to the study of the rough-wall case.

In this section we shall consider only the case of a dynamically completely rough wall, i.e. we shall assume that h_+ is high enough (namely $h_+ > 100$). It is known that in this case the mean velocity $U(y)$ above the wall protrusions does not depend on molecular viscosity. Hence the constant term B of the logarithmic equation for the mean velocity

$$U(y) = u_*(A \ln y_+ + B) \tag{5}$$

takes the form

$$B = -A \ln h_+ + B'(\sigma_1, \sigma_2, \ldots) = -A \ln (h_0 u_*/\nu), \tag{6}$$

where $h_0 = h \exp(-B'/A)$ is the so-called wall roughness parameter (see, e.g. Monin & Yaglom 1971, § 5.4). Equation (6) indicates that the momentum transfer from the flow to the wall is generated by the pressure differences between the fronts and backs of the protrusions (i.e. by their direct dynamical resistance), and not by viscous friction due to molecular viscosity. On the other hand the heat (or mass) exchange between any wall and fluid flow can be produced only by molecular diffusion; hence it is quite inadmissible to neglect the molecular diffusivity when the heat or mass transfer is considered. Correspondingly $\theta(y)$ will depend on χ even at $y \gg h$, i.e. β will depend on Pr.

As was mentioned above, we shall restrict ourselves to the case of $Pr \gtrsim 1$, i.e. we shall not consider heat transfer in rough-wall flows of liquid metals (since at present there are no experimental data on such heat transfer). Moreover we shall follow Levich (1962, § 30) and Owen & Thomson (1963) and assume that the roughness elements are closely spaced so that their sizes and shapes fully determine the flow in gaps between adjacent protrusions (forming the main part of the wall area). It was suggested by Levich that the order of magnitude of the thickness δ_v of the viscous sublayer in the gaps can be roughly estimated under these conditions with the aid of the following reasoning. Since it is supposed that the flow between wall protrusions and above their tops is fully determined by the protrusions themselves, the flow at a y of the order h is characterized by the length scale h and the ordinary velocity scale u_*. Hence the mean velocity $U(y)$ at

† The value $\beta_1 \approx 0{\cdot}5$ was in fact chosen by the authors in their previous work to achieve a better agreement with the heat transfer measurements in liquid metals, which will not be considered in the present paper.

the height $y = h$ is of order u_* and the eddy viscosity ϵ_M in the upper part of gaps between the wall protrusions is proportional to $u_* h$. Moreover,

$$\epsilon_M dU/dy = u_*^2,$$

hence $U(y) \sim u_* y/h$ in the upper part of the gaps. Now the thickness δ_v of the viscous sublayer can be determined by the usual condition that the corresponding Reynolds number $Re_{\delta_v} = \delta_v U(\delta_v)/\nu$ is of order unity. This leads to Levich's result

$$\delta_v \sim (\nu h/u_*)^{\frac{1}{2}} = h/h_+^{\frac{1}{2}}, \tag{7}$$

which agrees also with the relation given by the way on p. 333 of Owen & Thomson (1963).

The molecular effects are of no importance above the viscous sublayer (i.e. at $y > \delta_v$) in the case $Pr \gtrsim 1$ being examined. Therefore the dependence of β on Pr must be fully determined by the value of the difference $\theta_w - \theta(\delta_v)$. The accurate evaluation of this difference requires a physically realistic and mathematically treatable detailed model of the fluid motion in the gaps between the wall protrusions but the construction of such a model seems to be an impossible task. (Qualitative descriptions of the motion were proposed, for example, by Owen & Thomson (1963) and Dipprey & Sabersky (1963) and they are consistent with the arguments below.) This is apparently the reason why Owen & Thomson (1963) and Jayatilleke (1969), who needed the value of $\theta_w - \theta(\delta_v)$ in their studies, did not try to determine it theoretically, but used a simple empirical equation of the form

$$\theta_w - \theta(\delta_v) = b/\theta_* h_+^p Pr^q. \tag{8}$$

Here b, p and q are numerical constants (p and q are universal but b may depend on the shapes and distribution of the protrusions, i.e. on $\sigma_1, \sigma_2, \ldots$). The values of these constants were found in both the above studies from the analysis of inevitably limited and rather scattered experimental data; according to Owen & Thomson $p = 0{\cdot}45$ and $q = 0{\cdot}8$, while according to Jayatilleke $p = 0{\cdot}359$ and $q = 0{\cdot}695$. However the thickness of the sublayer relevant to equation (8) was not stated clearly enough in these studies and the attempt at physical interpretation of their values of p and q in the closing section of Owen & Thomson (1963) does not seem to be correct (it is based on the implausible assumption that h and u_* are appropriate length and velocity scales not only in the upper part of the gaps but also in the sublayer of molecular transfer). Therefore we propose another approach to the rough estimation of the form of the dependence of $\theta_w - \theta(\delta_v)$ on h_+ and Pr. This approach leads to an equation which is related to (7) with quite definite values of p and q and agrees reasonably well with all the available experimental data.

The subsequent arguments do not require knowledge of the details of the fluid motion in the gaps between wall protrusions but are based on more formal considerations but forward by Levich (1962, chap. 3) and also used by Kader & Yaglom (1970, 1972) for the analysis of turbulent heat and mass transfer at smooth walls. The main point is that all the eddy diffusivities (i.e. the eddy viscosity ϵ_M and eddy diffusivity for heat or matter ϵ_H) are equal to zero on the wall and must decrease as a power of y (let us say as y^m) when $y \to 0$. From the continuity equation we must have $m \geqslant 3$ (cf. Monin & Yaglom 1971, § 5.3). Since there are no physical reasons implying that $m > 3$ it is natural to assume that $m = 3$ and such an assumption is also confirmed by almost all recent experimental data (see Monin & Yaglom 1971, § 5.7; Kader & Aronov 1970). Therefore we shall assume that $\epsilon_M = a_M \nu y_+^3$ and $\epsilon_H = a_H \nu y_+^3$, where a_M and a_H are numerical coefficients. a_M and a_H are universal constants in smooth-wall flows and they determine the thickness of the viscous sublayer and the sublayer of molecular diffusion since $\epsilon_M = \nu$ and $\epsilon_H = \chi$ at the upper boundary of these sublayers (according to Kader & Yaglom (1972) both a_M and a_H are close to 0·001 over a smooth wall). However, in the case of a completely rough wall it is necessary to take into account the fact that wall protrusions significantly decelerate the flow in gaps. Therefore the velocity in the gaps must be appreciably smaller and the turbulent exchange appreciably weaker than at the same height above a smooth wall. This leads to the conclusion that a_M and a_H must depend on h_+ and decrease with increasing h_+ in the case of a rough wall. It seems natural to suppose that ϵ_M and ϵ_H are of the same order and proportional to each other since both of them characterize the vertical turbulent transfer produced by fluctuations of dynamical origin. Then it is possible to estimate the dependence of a_M and a_H on h_+ from the requirement that both ϵ_M and ϵ_H be of order ν at the height δ_v. It follows from this that $a_M \sim h_+^{-\frac{3}{2}}$ and $a_H \sim h_+^{-\frac{3}{2}}$. Hence $\epsilon_H = a'_H \nu h_+^{-\frac{3}{2}} y_+^3$ for $y < \delta_v$, where a'_H does not depend on h_+ (but may depend on the parameters $\sigma_1, \sigma_2, \ldots$, describing the shapes and the distribution of the protrusions).

If $Pr \approx 1$ then χ dominates the eddy diffusivity ϵ_H in the whole layer $0 < y < \delta_v$ (or at least in the main part of this layer) and therefore the order of the difference $\theta_w - \theta(\delta_v)$ can be estimated by solving the molecular diffusion equation

$$-c_p \rho \chi \, d\theta/dy = j_w$$

for $0 < y < \delta$. This leads to the conclusion that $\theta_w - \theta(\delta_v) \sim \theta_* h_+^{\frac{1}{2}}$ when $Pr \approx 1$, which was also found in the same way by Owen & Thomson (1963, p. 333). However the analysis of the situation when Pr is significantly different from unity (i.e. the estimate of $\theta_w - \theta(\delta_v)$ for the limiting case of $Pr \gg 1$) cannot be achieved without taking into account the turbulent flux determined by the eddy diffusivity ϵ_H. If $Pr \gg 1$ then the molecular diffusivity dominates the eddy diffusivity only within a thin sublayer of molecular transfer. The thickness δ_m of this sublayer is determined by the condition $\epsilon_H(\delta_m) = \chi$, i.e.

$$a'_H \nu h_+^{-\frac{3}{2}} (\delta_m u_*/\nu)^3 = \chi \quad \text{and} \quad \delta_m \sim h/h_+^{-\frac{1}{2}} Pr^{-\frac{1}{3}} = \delta_v Pr^{-\frac{1}{3}}.$$

However, at $\delta_m < y < \delta_v$ the eddy diffusivity dominates the molecular diffusivity. Hence to estimate $\theta_w - \theta(\delta_v)$ we must either neglect the eddy diffusivity within the molecular diffusivity sublayer and the molecular diffusivity above it and assume that $-\chi d\theta/dy = j_w/c_p\rho$ for $0 < y < \delta_m$ and $-\epsilon_H d\theta/dy = j_w/c_p\rho$ for $\delta_m < y < \delta_v$, or follow Kader (1966) and assume that $-(\chi + \epsilon_H) d\theta/dy = j_w/c_p\rho$ for the whole layer $0 < y < \delta_v$. Both these assumptions lead to the same result of the form

$$\theta_w - \theta(\delta_v) = \theta_* h_+^{\frac{1}{2}}/(b'_1 Pr^{\frac{2}{3}} - b'_2), \tag{9}$$

where b_1' and b_2' are numerical coefficients which may depend on $\sigma_1, \sigma_2, \ldots$ (cf. the similar derivation of the smooth-wall equation $\beta \approx b_1 Pr^{\frac{2}{3}} - b_2$ in the paper by Kader & Yaglom (1972), where it is also shown that available experimental data agree well with this equation if $b_1 = 12{\cdot}5$ and $b_2 = 6$).† Equation (9) agrees with the result that $\theta_w - \theta(\delta_v) \sim \theta_* h_+^{\frac{1}{4}}$ when $Pr \approx 1$. It suggests an approximate dependence of $\theta_w - \theta(\delta_v)$ on Pr and h_+ for all $Pr \gtrsim 1$ and $h_+ > 100$ which does not differ too much from the empirical equations of the form (8) proposed by Owen & Thomson (1963) and by Jayatilleke (1969).

Above the viscous sublayer (i.e. at $y > \delta_v$) we may neglect the molecular diffusivities ν and χ, and the eddy diffusivities ϵ_M and ϵ_H are here of the same order. Hence the vertical variation of the mean temperature in the layer above the viscous sublayer will be approximately proportional to the vertical variation of the mean velocity. If we assume that the turbulent Prandtl number $Pr_t = \epsilon_M/\epsilon_H$ within the entire buffer layer between the upper boundary of the viscous sublayer and the lower boundary of the logarithmic region takes the same constant value $\alpha A = \alpha/k \approx 0{\cdot}85$ as it takes within the logarithmic region, then we find that the variation of the dimensionless temperature $\theta_+(y) = [\theta_w - \theta(y)]/\theta_*$ in the layer from $y = \delta_v$ until any height y in the logarithmic region differs from the corresponding variation of the dimensionless velocity $U_+(y) = U(y)/u_*$ by only the constant factor α/A. Having combined this fact with (9) we obtain for the constant term β in the logarithmic temperature equation the following expression:

$$\beta = h_+^{\frac{1}{4}}(b_1' Pr^{\frac{2}{3}} - b_2') - \alpha(\ln h_+ - B'/A), \tag{10}$$

where $A = 1/k$ and B' have the same meaning as in (5) and (6). However there is in fact no reason to suppose that Pr_t is strictly constant within the entire buffer layer and logarithmic region; this last circumstance leads to the replacement of (10) by the slightly more general equation

$$\beta = h_+^{\frac{1}{4}}(b_1' Pr^{\frac{2}{3}} - b_2') - \alpha \ln h_+ + C, \tag{10'}$$

where C is a new numerical coefficient which may depend on the parameters $\sigma_1, \sigma_2, \ldots$, and the combination AC/α may differ (but not too much) from the coefficient B' in (6).

If we substitute (10′) into (4) and take into account the fact that

$$Re(\tfrac{1}{2}c_f)^{\frac{1}{2}} = Lu_*/\nu \quad \text{and} \quad h_+ = hu_*/\nu,$$

† Let us emphasize that in (8) j_w denotes the real flux from a unit area of the wall (in the gaps between its protrusions), while heat engineers usually compute values of the heat transfer coefficient c_h using a value of j_w equal to the ratio of the total heat transfer to the total area of the corresponding smooth wall. Thus the replacement of j_w in (8) by $j_w' = j_w F_s/F_r$, where F_r is the real area of the rough wall and F_s is the area of the similarly located smooth wall, may seem to be reasonable. However, such a replacement is unnecessary since the ratio F_s/F_r is determined primarily by $\sigma_1, \sigma_2, \ldots$, and hence it can be included in the coefficients b_1' and b_2'. Moreover, the replacement is also not justified since it is only relevant when all the area elements of the wall are completely equivalent in relation to heat transfer. This last condition is clearly incorrect in the case of a rough wall and our arguments are related only to gaps between the wall protrusions, which are supposed to produce the main contribution to the total heat transfer.

we obtain the following equation for the coefficient of heat transfer from a completely rough wall:

$$c_h = \frac{(\frac{1}{2}c_f)^{\frac{1}{2}}}{\alpha \ln (L/h) + h_+^{\frac{1}{2}}(b_1' Pr^{\frac{2}{3}} - b_2') + C + \beta_1}. \tag{11}$$

In the next section we shall compare this equation with the available experimental data. At this point we merely note that at high enough Re and not too low Pr the term proportional to $h_+^{\frac{1}{2}}$ will play the leading role in the denominator of the right-hand side of (11). This term increases with increasing u_* (i.e. with increasing Re) and hence (11) implies that the heat transfer from a rough wall at high enough Re may turn out to be even smaller than the heat transfer from a smooth wall at the same values of Re and Pr. This last effect evidently must be more noticeable for high values of Pr. The conclusion obtained contradicts the usual view on the augmentation of heat and mass transfer caused by wall roughness (see, for example, Kays 1966, §9.18; Bergles 1969). A comparison of it with the data will be made at the end of the next section.

3. Analysis of the experimental data

The most direct method of comparing (10′) with the data and determining the values of the numerical coefficients in it that best fit the data is based on the analysis of measurements of temperature and concentration distributions in various developed turbulent flows along completely rough walls. The measured dimensionless profiles $\theta_+(y) = [\theta_w - \theta(y)]/\theta_*$ can be approximated by logarithmic equations and their constant terms $\beta = \theta_+(y) - \alpha \ln y_+$ can then be compared with the values implied by (10′). Unfortunately until now there has been a great lack of good-quality measurements of temperature and concentration distributions in turbulent rough-wall flows. In particular there are very few data which confirm reliably the validity of the logarithmic equation for the mean temperature or concentration profile near a rough wall. Moreover, the evaluation of $\theta_+(y)$ requires knowledge of the values u_* and j_w in addition to the values of $\theta(y)$, but values of u_* and j_w are not given in the most of the experimental studies. However the data of Chamberlain (1968) may be used for approximate determination of the coefficients entering (10′). Chamberlain measured the velocity profile and the profile of mean concentration of radioactive vapour of ThB or of ordinary water vapour together with the corresponding values of u_* and j_w in a number of laboratory wind-tunnel flows along several artificial completely rough surfaces of simple geometry. His paper contains data on the dimensionless velocity $U_+(5)$ and dimensionless concentration $\theta_+(5)$ at the height $y = 5$ cm belonging to the logarithmic region of the flow together with the values of the height of the wall protrusions h, roughness parameter $h_0 = h \exp(-B'/A)$ and friction velocity u_*. If, as usual, we assume that $A = 2{\cdot}5$ and $\alpha = 2{\cdot}12$ (so that $Pr_t = \alpha/A = 0{\cdot}85$), then by virtue of (1), (3), (5) and (6) we obtain

$$\theta_+(5) - 0{\cdot}85 U_+(5) + 2{\cdot}12 \ln (h/h_0) = \beta + \alpha \ln h_+. \tag{12}$$

Thus if (10′) is approximately valid then the experimental values of

$$\tilde{\beta} = \theta_+(5) - 0{\cdot}85 U_+(5) + 2{\cdot}12 \ln h/h_0$$

must be represented in the form of a sum $C_1 h_+^{\frac{1}{2}} + C$, where $C_1 = b_1' Pr^{\frac{2}{3}} - b_2'$. The values of $\tilde{\beta}$ computed from Chamberlain's data are in fact rather scattered, which is not surprising since the accuracy of the measurements was not high (Chamberlain himself noticed insufficient accuracy of the measurements of u_* and h_0). However, on the average the values of $\tilde{\beta}$ obtained prove to be described by an equation of the form $\tilde{\beta} = C_1 h_+^{\frac{1}{2}} + C$ with a satisfactory accuracy for all values of $h_+ = hu_*/\nu$ from 10^2 to 4×10^3 and all the experimental surfaces sufficiently closely covered by roughness elements. The types of such 'close-packed' roughness used include the following arrangements: (*a*) spheres of diameter 2·54 or 0·79 cm placed on a flat plate at the vertices of a rather dense regular lattice, (*b*) cylinders of diameter 2·54, 0·79 or 0·16 cm with their axes perpendicular to the stream direction placed regularly and rather densely, (*c*) half-cylinders of diameter 5·08 cm placed on a plate perpendicular to the stream direction and close to each other, and (*d*) wavelike forms of height 0·6 and 5·7 cm and wavelength about ten heights covering the entire plate. Evaluation of the coefficients C and C_1 for different types of roughness and two different diffusing substances (ThB vapour and water vapour) shows that the values of C do not vary too much and that the estimate $C \approx 9·5$ can be considered in all the cases as more or less satisfactory. On the other hand the values of C_1 are rather insensitive to changes in the underlying rough surface but vary significantly and quite regularly when the diffusing substance is changed: the values of C_1 for ThB vapour (diffusion Prandtl number $Pr = 2·77$) are all considerably higher than the corresponding values for water vapour ($Pr = 0·62$). This fact clearly agrees with the equation given above describing the form of the dependence of C_1 on Pr. It turns out that the particular relation $C_1 = 0·55(Pr^{\frac{2}{3}} - 0·2)$ precisely fitting this equation describes with sufficient accuracy all the data on $\tilde{\beta}$ yielded by Chamberlain's measurements. Hence we see that according to the data considered the values of the numerical coefficients b_1', b_2' and C in (10′) prove to be approximately constant for a number of quite different types of roughness, although theoretically they may depend upon the parameters $\sigma_1, \sigma_2, \ldots$, describing specific features of the surface geometry. On the basis of this important circumstance we may suggest the single equation

$$\beta = 0·55 h_+^{\frac{1}{2}}(Pr^{\frac{2}{3}} - 0·2) - 2·12 \ln h_+ + 9·5 \tag{13}$$

as the first approximation to the constant term β of the logarithmic equation describing the mean temperature or concentration distribution at $Pr \gtrsim 1$ in turbulent flows along a great variety of different rough surfaces covered with closely spaced roughness elements. Let us also note that the suggested value 9·5 of the coefficient C in (10′) agrees satisfactorily with the prediction of its relative closeness to the value of the combination $\alpha B'/A$. In fact, if the value $B' = 8·5$ obtained by Nikuradse for the case of homogeneous sand roughness is taken as a typical value of B' then $\alpha B'/A \approx 0·85B' \approx 7·25$ and this number has the same order of magnitude as the suggested value of C.

A comparison of the measured values $\tilde{\beta}_m$ of $\tilde{\beta} = \beta + 2·12 \ln h_+$ derived from Chamberlain's data with the corresponding calculated values $\tilde{\beta}_c$ obtained with the aid of (13) is shown in figure 1. We have also plotted in figure 1 two additional

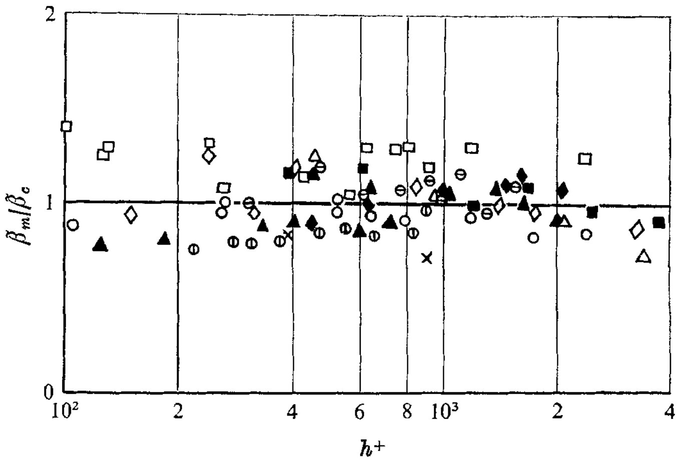

FIGURE 1. Ratio $\tilde{\beta}_m/\tilde{\beta}_c$ as a function of h_+. Chamberlain's data for various types of roughness: ○, spheres; □, ■, cylinders; △, ▲, half-cylinders; ◇, ◆, waves (open symbols refer to diffusion of ThB vapour and closed symbols to diffusion of water vapour). Results derived from Owen & Thomson's data: ⦶, two-dimensional roughness; ⊖, three-dimensional roughness. ×, results of Becirspahic.

points derived from the data of Becirspahic (1969, 1971) in order to increase the variety of flows used for comparison with (13). These data consist of the results of mean velocity and temperature profile measurements and measurements of u_* and j_w in two turbulent air flows (at $Pr = 0{\cdot}71$) in a rectangular channel, with one wall covered with two-dimensional strip roughness of trapezoidal form. Despite the fact that the experimental conditions, the value of Pr and the type of roughness were here quite different from those for Chamberlain's measurements, one can see that the corresponding values of $\tilde{\beta}_m/\tilde{\beta}_c$ do not differ very significantly from one. Finally, we plotted in figure 1 the points derived implicitly from the mass transfer data of Owen & Thomson (1963). These authors measured the rate of vertical mass transfer of camphor in air boundary layers along two horizontal rough plates sprayed with a camphor solution. In these experiments the diffusion Prandtl number was equal to 3·2 and the roughness elements consisted either of irregularly scattered pyramids or of regular spanwise humps forming two-dimensional roughness. Owen & Thomson did not include any direct data on camphor concentration profiles, but they suggested an indirect method of estimation of the vertical concentration differences in the immediate proximity of the wall based on the use of Reynolds' analogy (i.e the assumption that $Pr_t = 1$ everywhere). The same method can also be applied to derive the approximate values of β and $\tilde{\beta} = \beta + 2{\cdot}5 \ln h_+$ for Owen & Thomson's experiments and these approximate values are used as $\tilde{\beta}_m$ values to be compared with $\tilde{\beta}_c$ values in figure 1. (We now write $2{\cdot}5 \ln h_+$ instead of $2{\cdot}12 \ln h_+$ in the definition of $\tilde{\beta}$ because the method of evaluation of β used is based on the assumption that $\alpha = A = 2{\cdot}5$ and $Pr_t = \alpha/A = 1$.) Of course this treatment of Owen &Thomson's data is rather crude, but the lack of enough more accurate measurements of β justifies the inclusion of these data in figure 1. Although the scatter of points in figure 1 is considerable, the values of $\tilde{\beta}_m/\tilde{\beta}_c$ presented clearly cluster near one for all the values of Pr and all the types of roughness.

Since β enters the general equation (4) for the heat or mass transfer coefficient c_h, the data on heat and mass transfer from a rough wall can also be used to verify (13). Such a possibility has been in fact already used in figure 1 in relation to boundary-layer mass transfer data of Owen & Thomson. Now we shall exploit it more systematically in relation to numerous published studies of heat transfer in rough pipes. We must take into consideration, however, the fact that all the existing data on such a transfer concern the heat transfer coefficients defined in such a manner that the bulk velocity

$$U_b = \frac{2}{L^2}\int_0^L (L-y)\,U(y)\,dy$$

and the bulk temperature

$$\theta_b = \frac{2}{U_b L^2}\int_0^L (L-y)\,\theta(y)\,U(y)\,dy$$

(where L is a pipe radius) are used as velocity and temperature scales. In other words all the available information is related to the numbers

$$C_h = j_w/c_p\rho U_b(\theta_w-\theta_b)$$

or $Nu = C_h\,Re\,Pr$ and not to the numbers $c_h = j_w/c_p\rho U(\theta_w-\theta_1)$ considered above in this paper. Since the velocity scale U in (4) and (11) may be chosen quite arbitrarily provided that the same scale is used in dimensionless combinations $Re = UL/\nu$ and $c_f = 2(u_*/U)^2$, we can suppose from the very beginning that $U = U_b$. The transformation from c_h to C_h will be accomplished under such a supposition by multiplication by

$$\frac{1}{\Delta} = \frac{\theta_w-\theta_1}{\theta_w-\theta_b}, \quad \text{where} \quad \theta_w-\theta_b = \frac{2}{U_b L^2}\int_0^L (L-y)\,[\theta_w-\theta(y)]\,U(y)\,dy. \tag{14}$$

Since the fluid filling gaps between protrusions contributes very slightly to both the bulk velocity and the difference $\theta_w-\theta_b$, it can be neglected in the first approximation when calculating U_b and $\theta_w-\theta_b$. This means that the integration from $y=0$ to $y=L$ in the definition of U_b and $\theta_w-\theta_b$ can be approximately replaced by integration from $y=h$ to $y=L$. Moreover, we can use the fact that the velocity and temperature defect laws are valid within almost all the region $h<y<L$ with the exception only of a thin region near the tops of the wall protrusions whose contribution to the bulk values of velocity and temperature is also practically negligible. Hence we can assume that these laws are valid for all y between h and L without introducing a considerable error. In other words, we can assume that

$$\theta_w-\theta(y) = \theta_w-\theta_1-\theta_*\phi_1(y/L), \quad U(y) = U_1-u_*f_1(y/L) \tag{15}$$

for $h<y<L$, where U_1 is the mean velocity on the pipe axis and $f_1(y/L)$ the universal velocity-defect function. Under such assumptions we easily obtain

$$U_b = U_1(1-\eta_1)^2-2u_*\int_{\eta_1}^1 (1-\eta)f_1(\eta)\,d\eta, \tag{16}$$

$$\theta_w-\theta_b = \theta_w-\theta_1-2\theta_*\frac{U_1}{U_b}\int_{\eta_1}^1 (1-\eta)\,\phi_1(\eta)\,d\eta+2\theta_*\frac{u_*}{U_b}\int_{\eta_1}^1 (1-\eta)\,\phi_1(\eta)f_1(\eta)\,d\eta, \tag{17}$$

where $\eta_1 = h/L$ is a relative roughness height. For the approximate evaluation of the integrals on the right-hand sides of (16) and (17) we can use the fairly accurate assumption that the logarithmic form of both defect laws is valid to the pipe axis. In other words, we can in a first approximation use the relations $f_1(\eta) = A\ln\eta$ and $\phi_1(\eta) = \alpha\ln\eta$ (the necessary conditions $f_1(1) = \phi_1(1) = 0$ are here evidently valid). If we now replace the integration from $\eta = \eta_1$ to $\eta = 1$ on the right-hand sides of (16) and (17) by integration from $\eta = 0$ to $\eta = 1$, additional errors which are of the same order as the already neglected contributions to U_b and θ_b appear. Therefore it is possible to extend the integrals on the right-hand sides of (16) and (17) from 0 to 1 without changing appreciably the accuracy of the equations. This extension together with the use of a logarithmic approximation to the functions $f_1(\eta)$ and $\phi_1(\eta)$ leads to the following results:

$$U_b = U_1(1-\eta_1)^2 - 1{\cdot}5Au_* \tag{18}$$

and

$$\theta_w - \theta_b = \theta_w - \theta_1 - 1{\cdot}5\alpha\frac{U_1}{U_b}\theta_* + 3{\cdot}5\alpha A\frac{u_*}{U_b}\theta_*. \tag{19}$$

These two equations imply the relation

$$\Delta = \frac{\theta_w-\theta_b}{\theta_w-\theta_1} = 1 - \frac{\theta_*}{\theta_w-\theta_1}\left[\frac{1{\cdot}5\alpha}{(1-\eta_1)^2} - \left(3{\cdot}5 - \frac{2{\cdot}25}{(1-\eta_1)^2}\right)\alpha A(\tfrac{1}{2}c_f)^{\frac{1}{2}}\right], \tag{20}$$

where $c_f = 2(u_*/U_b)^2$. Since $\theta_*/(\theta_w-\theta_1) = j_w/c_p\rho u_*(\theta_w-\theta_1) = c_h(c_f/2)^{-\frac{1}{2}}$, equation (11) for c_h and equation (20) for the correction factor Δ lead to an approximate equation for C_h of the form

$$C_h = \frac{(\frac{1}{2}c_f)^{\frac{1}{2}}}{\alpha\ln\dfrac{L}{h} + (b_1'Pr^{\frac{2}{3}} - b_2')h_+^{\frac{1}{2}} + C + \beta_1 - \dfrac{1{\cdot}5\alpha}{(1-\eta_1)^2} + \left[3{\cdot}5 - \dfrac{2{\cdot}25}{(1-\eta_1)^2}\right]\alpha A(\frac{1}{2}c_f)^{\frac{1}{2}}}. \tag{21}$$

In accordance with the considerations presented in § 1 we shall suppose that $A = 2{\cdot}5$, $\alpha = 2{\cdot}12$ and $\beta_1 = 0{\cdot}5$. Moreover we shall use (13), which implies that $b_1' = 0{\cdot}55$, $b_2' = 0{\cdot}55\times 0{\cdot}2 = 0{\cdot}11$ and $C = 9{\cdot}5$. We shall also simplify (21) by taking into account the fact that the terms of the denominator of its right-hand side containing the factor $(\frac{1}{2}c_f)^{\frac{1}{2}}$ prove to be rather small compared with the other terms in all ordinary situations. Hence it is justifiable to use a simplified estimate of these terms instead of their true values. Let us now note that η_1 is usually small in comparison with 1 and therefore the replacement of $(1-\eta_1)^2$ by 1 in the expression for the coefficient of $(\frac{1}{2}c_f)^{\frac{1}{2}}$ changes this coefficient only slightly. For this reason preservation of the factor $(1-\eta_1)^{-2}$ in the considered expression seems quite unjustified. (In the case of a smooth-wall flow the factor $(\frac{1}{2}c_f)^{\frac{1}{2}}$ is still smaller and therefore the term containing this factor was neglected altogether in the corresponding equation of the paper by Kader & Yaglom (1972).) Substitution of the above-mentioned numerical values of the coefficients entering (21) and replacement of $2{\cdot}25/(1-\eta_1)^2$ by the simple constant $2{\cdot}25$ lead to the following result:

$$Nu = C_h Re\,Pr = \frac{(\frac{1}{2}c_f)^{\frac{1}{2}} Re\,Pr}{2{\cdot}12\ln\dfrac{L}{h} + 0{\cdot}55\,(Pr^{\frac{2}{3}} - 0{\cdot}2)\,h_+^{\frac{1}{2}} + 10{\cdot}0 - \dfrac{3{\cdot}2}{(1-\eta_1)^2} + 6{\cdot}6(\frac{1}{2}c_f)^{\frac{1}{2}}}. \tag{22}$$

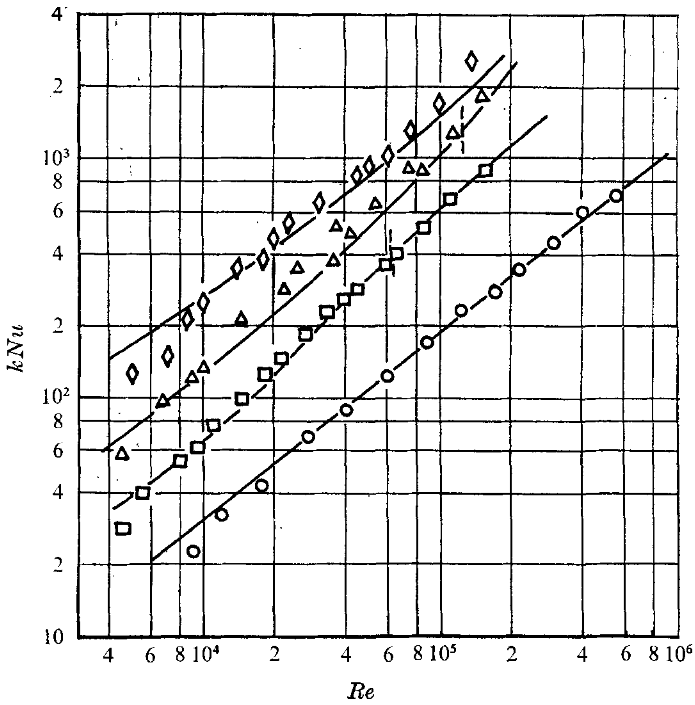

FIGURE 2. *Nu* as a function of *Re* according to Pinkel's data. Values of the product *kNu* with different *k* are plotted in order to avoid the mixing of data points. ——, proposed theoretical equation. $Pr = 0{\cdot}71$.

	○	□	△	◇
η_1	0	0·026	0·0375	0·017
Range of h_+	0	5–157	6–207	2–59
Range of h_{s+}	0	8–242	4–136	1–20
k	1	2	4	8

Now we shall compare this result with the data available on Nusselt numbers for heat transfer in rough pipes.

The experimental data suitable for such a comparison can be found in the studies by Pinkel (1954), Seleznev (1955, 1956), Nunner (1956), Teverovskii (1956, 1958), Dipprey & Sabersky (1963), Kolář (1965), Isachenko, Agababov & Galin (1965), Galin (1966) and Antuf'ev (1966). The roughness of the internal surface of the pipe was established in the corresponding experiments mainly by threading the surface, i.e. cutting helical threads of various forms (Pinkel; Teverovskii; Kolář; Galin; Antuf'ev). In some other cases the surface was roughened by circumferential rings (Nunner) or some three-dimensional protrusions (Selesnev; Nunner; Dipprey & Sabersky). The Prandtl number was close to 0·71 in the case of the experiments on heat transfer in air made by Pinkel, Seleznev, Nunner, Teverovskii and Antuf'ev, and it was varied from 1·2 to 9 depending on the temperature in the measurements of Dipprey & Sabersky, Isachenko *et al.* and Galin, who used water as the working fluid. Finally, Kolář used both air and water in his experiments.

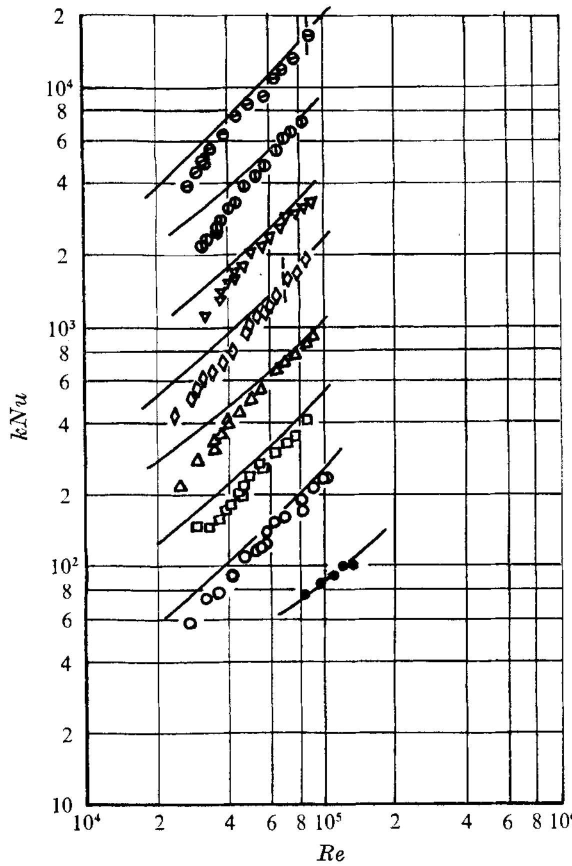

FIGURE 3. Nu as a function of Re according to Seleznev's data, plotted as in figure 2. Pr = 0·71.

	●	○	□	△	◇	▽	⦶	⊖
η_1	0	0·028	0·033	0·042	0·053	0·046	0·042	0·043
Range of h_+	0	26–94	34–95	38–131	48–171	53–133	47–124	45–139
Range of h_{s+}	0	13–48	21–58	19–67	33–118	21–54	28–74	32–100
k	$\frac{1}{2}$	1	2	4	8	16	32	64

A comparison of the measured values of Nu with the values calculated with the aid of (22) is shown in figures 2–7 for part of Pinkel's data (corresponding to relatively small temperature differences between wall and fluid) and data of Seleznev, Nunner, Teverovskii, Dipprey & Sabersky and Antuf'ev. To be sure that the used data relate in fact to completely rough flow conditions, the values of the friction coefficient at high enough Re were used to calculate the height of the equivalent (i.e. producing the same friction) sand roughness h_s. Then (22) was applied only when $h_{s+} = h_s u_*/\nu \geqslant 100$. Unfortunately it was found that many of the data refer to transitional flows with $h_{s+} < 100$. In an attempt to use also the transitional data we evaluated $\beta = \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots)$ for such flows by means of linear interpolation between dynamically smooth and dynamically completely rough walls. In other words we used the following equation:

$$\beta = \tfrac{1}{100} h_{s+} \beta_r + (1 - \tfrac{1}{100} h_{s+}) \beta_s, \tag{23}$$

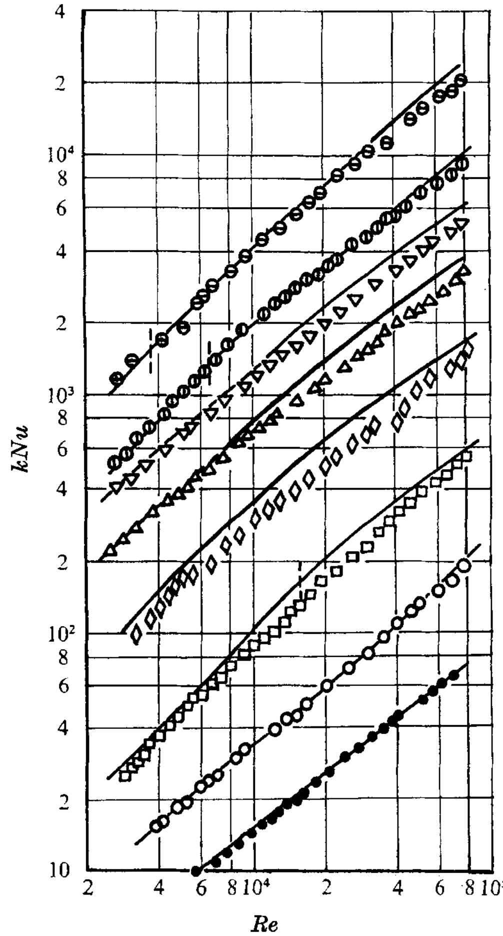

FIGURE 4. *Nu* as a function of *Re* according to Nunner's data, plotted as in figure 2. $Pr = 0{\cdot}71$.

	●	○	□	◇	△	▽	⦶	⊖
η_1	0	0·016	0·182	0·167	0·164	0·162	0·0805	0·0805
Range of h_+	0	2–41	26–701	45–1158	42–1230	37–1020	12–366	14–401
Range of h_{s+}	0	2–41	18–485	190–4900	239–7010	137–4110	38–1160	70–1950
k	$\frac{1}{2}$	1	2	4	8	16	32	64

where β_r is given by equation (13) and $\beta_s = 12{\cdot}5Pr^{\frac{2}{3}} - 6$ in accordance with the results of Kader & Yaglom (1972). [A similar method of determination of β values for transitional flows was used by Chamberlain (1968) but with another choice of equations for β_r and β_s. Another slightly more complicated method of determination of β for transitional flows was suggested by Jayatilleke (1969).] In the rare cases when the value of c_f for the transitional flow considered was not reported by the author, this value was also determined by linear interpolation between smooth and completely rough values of c_f. Values of *Nu* for transitional flows calculated from the data on β and c_f in accordance with (4) are also shown in figures 2–7. Vertical dotted lines in all the diagrams mark the boundary value

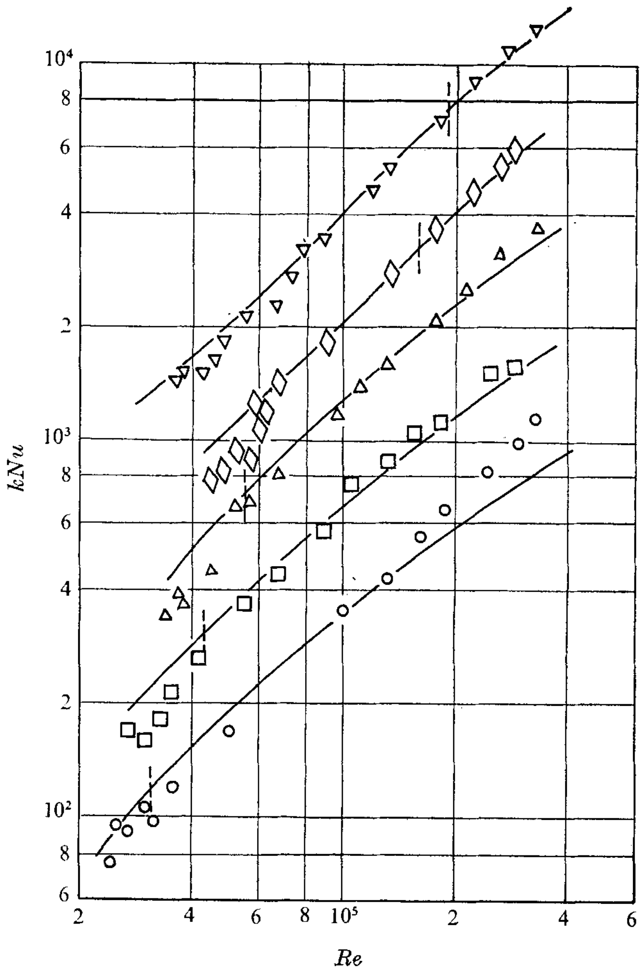

FIGURE 5. Nu as a function of Re according to Teverovskii's data, plotted as in figure 2. $Pr = 0{\cdot}71$.

	○	□	△	◇	▽
η_1	0·0785	0·0657	0·053	0·0267	0·0201
Range of h_+	82–1257	74–791	72–692	37–259	24–216
Range of h_{s+}	71–1080	60–635	62–592	26–181	19–170
k	1	2	4	8	16

of Re above which the pipe may be considered completely rough. When the experiments considered also included heat transfer measurements in a smooth pipe the corresponding values of Nu are also shown in the figures together with the theoretical curve implied by (4) with $\beta = \beta_s$. These last graphs allow us to compare the deviations of the measured values of the Nusselt number for rough-wall heat transfer from the corresponding calculated values of Nu with the analogous deviations in the case of a smooth pipe. The data of Kolář, Isachenko *et al.* and Galin are related to flows with varying Pr and are inconvenient for presentation in the form of diagrams similar to those in figures 2–7. Instead,

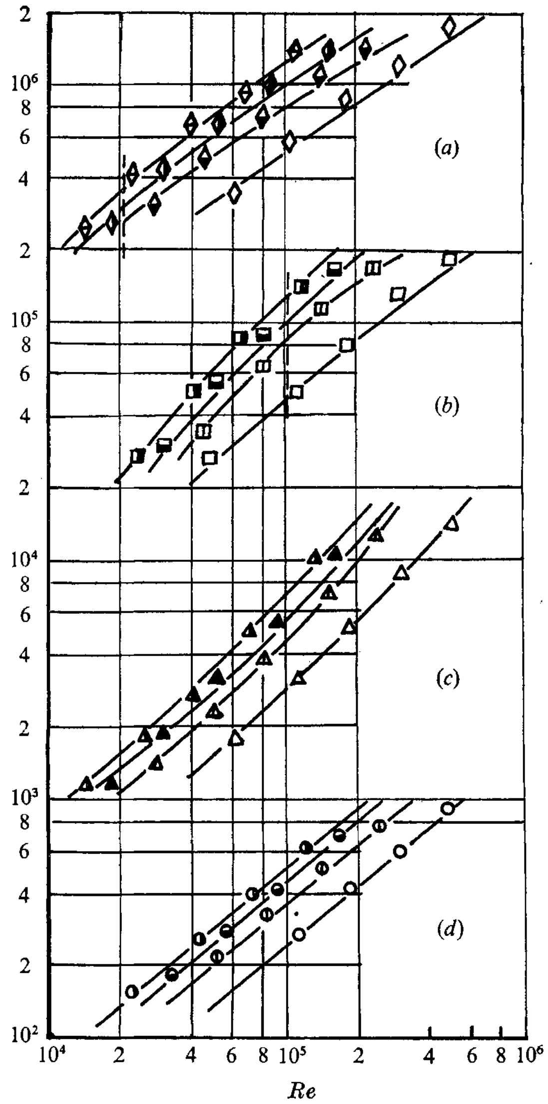

FIGURE 6. Nu as a function of Re according to Dipprey & Sabersky's data, plotted as in figure 2. (*a*) $k = 10^3$, $\eta_1 = 0{\cdot}0976$. (*b*) $k = 10^2$, $\eta_1 = 0{\cdot}0276$. (*c*) $k = 10$, $\eta_1 = 0{\cdot}0048$. (*d*) $k = 1$, $\eta_1 = 0$.

		◈	◈	◈	◇
(*a*)	Pr	5·94	4·38	2·79	1·20
	Range of $h_+ = h_{s+}$	65–510	83–695	175–1020	273–2381
		◨	⬓	◫	□
(*b*)	Pr	5·94	4·38	2·79	1·20
	Range of $h_+ = h_{s+}$	23–118	30–166	46–233	49–514
		▲	▲	△	△
(*c*)	Pr	5·94	4·38	2·79	1·20
	Range of $h_+ = h_{s+}$	2–16	2–19	3–29	7–60
		◑	◒	⦶	○
(*d*)	Pr	5·94	4·38	2·79	1·20

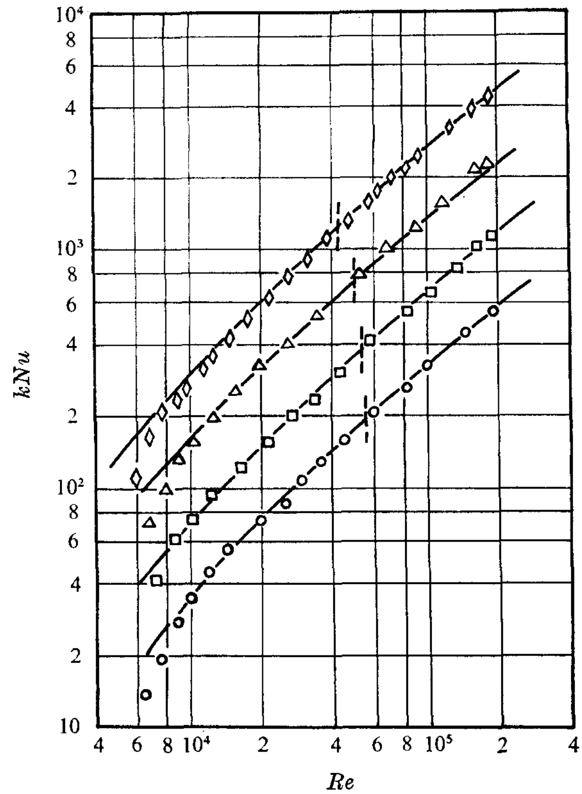

FIGURE 7. Nu as a function of Re according to Antuf'ev's data, plotted as in figure 2. $Pr = 0{\cdot}71$.

	○	□	△	◇
η_1	0·061	0·094	0·094	0·129
Range of h_+	16–453	28–693	26–692	48–991
Range of h_{s+}	12–335	14–350	14–367	21–436
k	1	2	4	8

figures 8 and 9 compare directly the corresponding measured values of Nu with the Nu calculated using (4), (22) and (23).

Figures 2–9 show that the agreement of the measured values Nu_m of the Nusselt number with the corresponding calculated values Nu_c is quite satisfactory in most cases in spite of the fact that the equations used do not include any dependence on the specific features of the roughness. Almost all the deviations of Nu_m from Nu_c do not exceed 10 %. Hence on the whole these figures confirm the applicability of the derived equations to the description of turbulent heat and mass transfer at rough walls for a wide range of roughness types and the values of h_+ and Pr.

The data of Seleznev (for several types of three-dimensional pyramidal roughness) show somewhat worse agreement with the calculation than most of the other data, but they related almost entirely to transitional flows and are in general exceptional in some senses. The data of Galin are especially scattered. This may be partially explained by the fact that the experiments on heat transfer

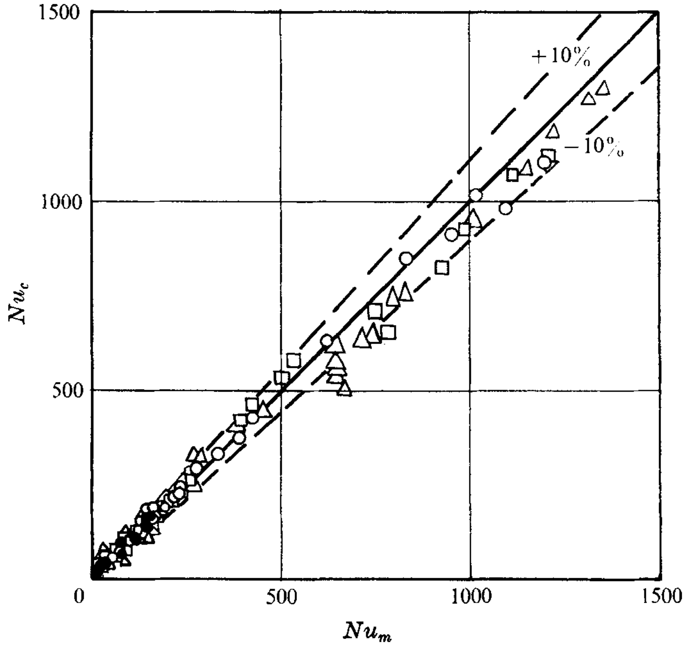

FIGURE 8. Comparison of measured values Nu_m of Nu with calculated values Nu_c for Kolář's data. ——, perfect agreement; – – –, 10 % deviation. Pr = 0·71 or 3.

	●	○	□	△
η_1	0	0·0394	0·074	0·109
Range of h_+	0	10–209	14–470	26–766
Range of h_{s+}	0	8–172	13–468	30–864

in liquids are considerably more complicated than those on heat transfer in air, which was studied in most of the other sources used. In fact the temperature variations over the protrusion surfaces and the related variability of Pr in the proximity of the wall may influence significantly heat tranfser in the case of a liquid flow but they are quite unimportant in the case of an air flow. Let us note in this connexion that Galin's data are uncorrected while Kolář (1965) introduced some crude approximate corrections for these effects in his water heat transfer data related to similar type of roughness. These corrections changed Kolář's data noticeably and led to a marked decrease of scatter in figure 8. Finally, the systematic departure from the calculation of most of Nunner's data for pipes with relatively sparse internal rings may indicate the non-universality of (13) and the dependence of β on wall geometry. It is worth mentioning in this connexion that the data related to the quite sparsely spaced roughness anyhow show quite evidently that (13) cannot be applied in all cases (see, for example the data of Webb, Eckert & Goldstein (1971) on heat transfer in pipes with repeated-rib roughness).

The dependence of β on the Prandtl number suggested by (13) needs of course an additional careful verification using data related to much greater values of Pr. The first results on turbulent heat and mass transfer at rough surfaces at high

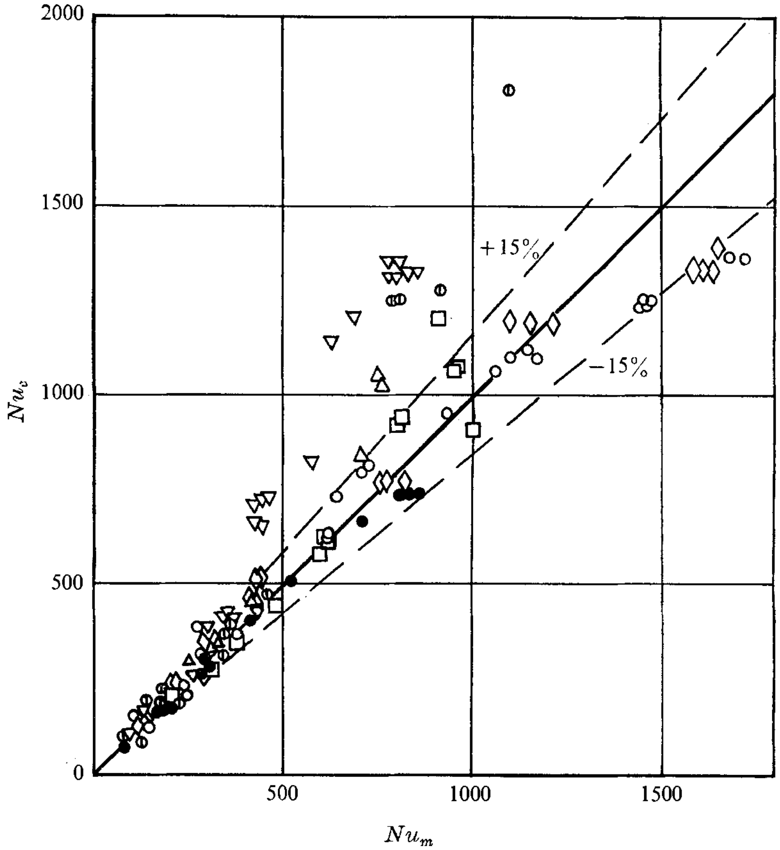

FIGURE 9. Comparison of measured values Nu_m of Nu with calculated values Nu_c for the data of Isachenko *et al.* (1965) and Galin (1966). ——, perfect agreement; – – –, 15% deviation. *Pr* varies from 6 to 9.

	●	○	□	△	◇	▽	⦶
η_1	0	0·712	0·218	0·125	0·878	0·055	0·011
Range of h_+	0	26–413	14–104	12–67	30–957	17–457	3–80
Range of h_{s+}	0	17–275	9–68	6–33	16–509	7–195	5–115

Prandtl number were published recently by Smith & Gowen (1965), Watson & Thomas (1967), Dawson & Trass (1972) and a few others. However, all the corresponding data are insufficiently full, refer to some specific conditions and do not allow a comparison with the equations proposed in our paper.

In conclusion let us consider the question of the double role of roughness in heat and mass transfer. It is usually supposed that turbulent heat and mass transfer is augmented by wall roughness in all the cases. However, we have already mentioned at the end of § 2 that heat and mass transfer at a rough wall should be weaker than at a smooth wall at the same values of *Re* and *Pr* according to the derived equations if *Re* is high enough. The point is that although the wall protrusions cause additional disturbances of the flow intensifying the transfer they also decelerate the flow near a wall. This deceleration causes reduction of heat and mass transfer and according to (11) and (22) it should even outweigh the intensifying effect of the additional disturbances if Re is very high. Careful inspection of Nunner's data, presented in figure 4, for a wide range of Re and h_+ values shows that in fact the slopes of the plotted rough-pipe curves exceed essentially the slope of the lowest smooth-pipe curve at not too high values of Re, but this difference of slopes decreases gradually with increasing Re. Moreover, in some cases the curves for rough pipes turn out to be even more slowly increasing than the curve for a smooth pipe at the greatest values of Re considered. Exactly the same effect can be noticed in some other graphs presented above, for example, in figure 6, illustrating Dipprey & Sabersky's data. It is very clearly evident also in some heat transfer diagrams of the paper by Isachenko *et al.* (1965). Especially expressive data were obtained by the last-mentioned authors in an annular water channel between two coaxial cylinders with a rough internal wall and a smooth external wall. It is clear that the equations above cannot be applied explicitly to heat transfer calculations for annular channels, but the qualitative reasons presented explain also the effect found (i.e. an essential excess of the rough-wall Nusselt number Nu_r over the corresponding smooth-wall value Nu_s at moderate Re which transforms into the opposite inequality $Nu_r < Nu_s$ at higher values of Re). A similar trend is noticeable in the graphs of Watson & Thomas (1967) and Dawson & Trass (1972) concerning the mass transfer of ions in electrolyte–water solutions (Pr of the order of several thousand). Here also at low and moderate values of Re the wall roughness caused an increase of the slope of the c_h *vs.* Re curve as compared with the value of the slope in the case of a smooth-wall flow; but at high values of Re the slope for a rough-wall flow was found to be even smaller than that for the case of a smooth wall. Finally, some of the figures included in the survey paper by Bergles (1969) are also qualitatively similar to those of Isachenko *et al.* (1965), Watson & Thomas (1967) and Dawson & Trass (1972). Thus the general conclusions on the possible deterioration of heat and mass transfer by the wall roughness at high enough Re (especially in the case of high Pr) implied by (11) and (22) are qualitatively confirmed by the available experimental data. However, reliable quantitative verification of this effect still requires special careful measurements of heat and mass transfer in rough- and smooth-walls flows at very high values of Re and Pr.

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